

## Mathematical Analysis

### **No calculators allowed on these problems.**

1. Write the general form (  $y = ax^2 + bx + c$  ) of the quadratic function that passes through the points  $(-4, 6)$ ,  $(-2, 7)$ , and  $(0, 6)$ .
2. Write the standard form of the quadratic function that has the vertex  $(-5, 1)$  and passes through the point  $(-2, 3)$
3. A farmer has 210 feet of fencing and wishes to enclose a rectangular plot of land for his pet Alpacas. The land is bordered on one side by a stream. He wants his Alpacas to have free access to the water in the stream. I am not sure how well Alpacas can jump, but suppose these ones hate jumping☺. Although the farmer does not want any fence along the stream, he would like them to have the maximum possible grazing area in their enclosure. Find the dimensions of this enclosure.
  - i) Show the relationship between the length  $x$  and width  $y$  of the enclosure.
  - ii) Write the area as an expression only in  $x$
  - iii) Use the equation that you found in part (ii) to find the required dimensions.
4.  $f(x) = -6x^4 + 5x^2 + 7x - 10$ . Use Descarte's rule of signs to complete the following table.

#of possible		
Positive zeros	Negative zeros	Imaginary zeros

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5. Write the quotient in standard form.
$$\frac{3 - 2i}{1 + 4i}$$
  6. Simplify  $i^{100} - i^{31} + i^7$
  7.  $f(x) = 2x^3 - 7x^2 + 22x + 13$ ;  $2 - 3i$  is a zero of  $f(x)$ . Find the remaining zeros.

8. Determine any intercepts, holes, vertical asymptotes, horizontal asymptotes, or slant asymptotes in the graph of  $y = \frac{x^2 - 9}{1 - x}$

y-intercept \_\_\_\_\_

x-intercept(s) \_\_\_\_\_

Coordinates of hole(s) \_\_\_\_\_

Equation(s) of V.A \_\_\_\_\_

Equation(s) of H.A \_\_\_\_\_

Equation(s) of S.A \_\_\_\_\_

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9. Determine any intercepts, holes, vertical asymptotes, horizontal asymptotes, or slant asymptotes in the graph of  $y = \frac{2x + 4}{x^2 - 4}$

y-intercept \_\_\_\_\_

x-intercept(s) \_\_\_\_\_

Coordinates of hole(s) \_\_\_\_\_

Equation(s) of V.A \_\_\_\_\_

Equation(s) of H.A \_\_\_\_\_

Equation(s) of S.A \_\_\_\_\_

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10. Write a rational function satisfying the following criteria:

Vertical asymptote:  $x = -5$

Slant asymptote:  $y = 2x - 1$

Zero to the function:  $x = 2$

Your answer should be in the form  $y = \frac{a \text{ polynomial}}{a \text{ polynomial}}$

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11. Find a quadratic function  $f(x) = ax^2 + bx + c$  such that the maximum value of  $f$  is 8 and the graph of  $f$  has x-intercepts 3 and 7.
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12. The difference of two numbers is 7. Find the minimum value of the product of these two numbers.

- i) Let  $x$  be the smaller number and  $p$  the product. Write  $p$  as a function of  $x$
- ii) Use the function from part (i) to solve the problem.

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13. Find a quadratic equation having roots  $\frac{1 \pm \sqrt{3}}{4}$ . The coefficients of your quadratic have to be integers.

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14. One of the zeros of  $f(x) = x^4 + 10x^3 + 26x^2 + 10x + 25$  is  $i$ . Find the remaining zeros.

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15. Find a polynomial function with real coefficients with zeros  $-4, 4, 1 + \sqrt{3}i$

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